LOGARITHM

Logarithmic Equations:

1. $\log_{x-1} 3 = 2$
2. $\log_4 (2\log_3 (1 + \log_2 (1 + 3\log_3 x))) = 1/2.$
3. $\log_3(1 + \log_3(2^x - 7)) = 1$
4. $\log_5\left(\frac{2+x}{10}\right) = \log_5\left(\frac{2}{x+1}\right)$
5. $1 + 2\log_{(x+2)}5 = \log_5(x + 2)$
6. $5^{\log_{10} x} = 50 - x^{\log_{10} 5}$
7. Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that $2(\log_a c + \log_b c) = 9\log_{ab} c$. Find the largest possible value of $\log_a b$.
8. Find the square of the sum of the roots of the equation
$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$.
9. Find the value of the expression $\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6}$.
10. Find a rational number which is 50 times its own logarithm to the base 10.
11. Calculate: (a) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$ (b) $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right) - 6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$
12. Simplify: $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \cdot \left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$.
13. Simplify: $5^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10 + 2\sqrt{21}}$.
14. Solve $\log_{5-x}(x^2 - 2x + 65) = 2$
15. Solve $\log_4 8 + \log_4(x+3) - \log_4(x-1) = 2$
16. Solve $\log_7 \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 0$
17. Solve $\log_3(3^x - 8) = 2 - x$
18. Solve $\log_2(25^{x+3} - 1) = 2 + \log_2(5^{x+3} + 1)$
19. Solve $\log_2(4 \times 3^x - 6) - \log_2(9^x - 6) = 1$
20. Solve $\log_4(2 \times 4^{x-2} - 1) + 4 = 2x$
21. Solve $6(\log_x 2 - \log_4 x) + 7 = 0$
22. Solve $3\log_x 4 + 2\log_{4x} 4 + 3\log_{16x} 4 = 0$
23. Solve $4^{\log_2 2x} = 36$

Logarithmic Inequalities:

24. $\log_{1/3} (5x - 1) > 0$
25. $\log_{1.5} \frac{2x-8}{x-2} < 0$
26. $\log_{1/4}\left(\frac{35 - x^2}{x}\right) \geq -\frac{1}{2}$
27. $\log_3 \frac{1+2x}{1+x} < 1$
28. Solve in R $\log_3(2x^2 + 6x - 5) > 1$
29. Solve $\log_{10}(x^2 - 2x - 2) \leq 0$
30. Solve $\log_{x-2}(3x^2 - x - 1) > 0$
31. Solve for $x \in R$ $\log_x\left(2x - \frac{3}{4}\right) > 2$
32. Solve $\log_2 \frac{x-4}{2x+5} < 1$
33. Solve $\log_{0.2} \log_6\left(\frac{x^2 - x}{x^2 + 1}\right) > 0$
34. Solve $\frac{\log_2(4x^2 - x - 1)}{\log_2(x^2 + 1)} > 1$
35. Solve $x^{(\log_{10} x)^2 - \log_{10} x^3 + 1} > 1000$
36. $\log_x (\log_9 (3^x - 9)) < 1$
37. $\log_x (x^3 - x^2 - 2x) < 3$
38. $\frac{2^{1-x} - 2^x + 1}{2^x - 1} \leq 0$
39. $\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} < \frac{1}{\log_4 (x+3)}$

ANSWER KEY

1. $\{1 + \sqrt{3}\}$
2. {3}
3. {4}
4. {3}
5. $\left\{-\frac{9}{5}, 23\right\}$
6. {100}
7. 2
8. 3721
9. $\frac{1}{6}$
10. N = 100
11. (a) 0 (b) 9
12. 1
13. 6
14. –5
15. 5
16. 4
17. 2
18. –2
19. 1
20. 2
21. $8, 2^{-2/3}$
22. $\frac{1}{2}, \frac{1}{8}$
23. 3
24. (1/5, 2/5)
25. (4, 6)
26. $[-7, -\sqrt{35}) \cup [5, \sqrt{35})$
27. $(-\infty, -2) \cup (-1/2, +\infty)$
28. $(-\infty, -4) \cup (-1, +\infty)$
29. $[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$
30. $(3, +\infty)$
31. $\left(\frac{3}{8}, \frac{1}{2}\right) \cup \left(1, \frac{3}{2}\right)$
32. $\left(-\infty, \frac{-14}{3}\right) \cup (4, +\infty)$
33. $(-\infty, -1)$
34. $\left(-\infty, \frac{-2}{3}\right) \cup (1, +\infty)$
35. $(1000, +\infty)$
36. $(\log_3 10, +\infty)$
37. $(2, +\infty)$
38. $(-\infty, 0) \cup (1, \infty)$
39. $(-1, \infty)$